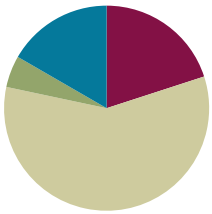
Lesson 29

Objective: Estimate sums and differences using benchmark numbers.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(3 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Count by Equivalent Fractions **4.NF.1** (6 minutes)
- Change Fractions to Mixed Numbers **4.NF.4** (6 minutes)

Count by Equivalent Fractions (6 minutes)

Note: This activity reviews Lesson 24. The progression builds in complexity. Build students to the highest level of complexity in which they can confidently participate.

T: Count by twos to 16, starting at 0.

S: 0, 2, 4, 6, 8, 10, 12, 14, 16.

T: Count by 2 fourths to 16 fourths, starting at 0 fourths. (Write as students count.)

$\frac{0}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{6}{4}$	$\frac{8}{4}$	$\frac{10}{4}$	$\frac{12}{4}$	$\frac{14}{4}$	$\frac{16}{4}$
0	$\frac{2}{4}$	1	$\frac{6}{4}$	2	$\frac{10}{4}$	3	$\frac{14}{4}$	4
0	$\frac{2}{4}$	1	$1\frac{2}{4}$	2	$2\frac{2}{4}$	3	$3\frac{2}{4}$	4

S: $\frac{0}{4}, \frac{2}{4}, \frac{4}{4}, \frac{6}{4}, \frac{8}{4}, \frac{10}{4}, \frac{12}{4}, \frac{14}{4}, \frac{16}{4}$.

T: 1 is the same as how many fourths?

S: 4 fourths.

T: (Beneath $\frac{4}{4}$, write 1.)

Continue the process for 2, 3, and 4.

T: Count by 2 fourths again. This time, when you come to the whole numbers, say the ones. Start at zero. (Write as students count.)

S: $0, \frac{2}{4}, 1, \frac{6}{4}, 2, \frac{10}{4}, 3, \frac{14}{4}, 4$.

T: (Point to $\frac{6}{4}$.) Say $\frac{6}{4}$ as a mixed number.

S: $1\frac{2}{4}$.

Continue the process for $\frac{10}{4}$ and $\frac{14}{4}$.

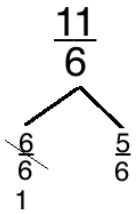
T: Count by 2 fourths again. This time, convert to whole numbers and mixed numbers. Start at zero. (Write as students count.)

S: $0, \frac{2}{4}, 1, 1\frac{2}{4}, 2, 2\frac{2}{4}, 3, 3\frac{2}{4}, 4$.

Change Fractions to Mixed Numbers (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 24.



T: (Write $\frac{11}{6}$.) Say the fraction.

S: 11 sixths.

T: (Draw a number bond with $\frac{11}{6}$ as the whole.) How many sixths are in 1?

S: 6 sixths.

T: (Write $\frac{6}{6}$ as a part. Write $\frac{\quad}{6}$ as the other part.) Write the unknown part.

S: (Write $\frac{5}{6}$ as the unknown part.)

T: (Cross out $\frac{6}{6}$, and write 1 beneath it. Write $\frac{11}{6}$ =____.) Write $\frac{11}{6}$ as a mixed number.

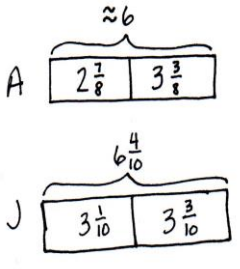
S: (Write $\frac{11}{6} = 1\frac{5}{6}$.)

Continue with the following possible sequence: $\frac{17}{6}, \frac{15}{4}$, and $\frac{29}{8}$.

Application Problem (3 minutes)

Both Allison and Jennifer jogged on Sunday. When asked about their distances, Allison said, "I ran $2\frac{7}{8}$ miles this morning and $3\frac{3}{8}$ miles this afternoon. So, I ran a total of about 6 miles," and Jennifer said, "I ran $3\frac{1}{10}$ miles this morning and $3\frac{3}{10}$ miles this evening. I ran a total of $6\frac{4}{10}$ miles."

How do their answers differ? Discuss with your partner.



Their answers differ because Allison estimated how far she ran. She said "about 6". Jennifer gave an exact answer because she added.

Note: This Application Problem prepares students for today's Concept Development by prompting them to think about and discuss exact answers and estimates. Therefore, student conversations should include exact and approximate reflections.

Concept Development (35 minutes)

Materials: (T) 4-inch piece of string (S) Personal white board

Problem 1: Estimate the sum or difference of two mixed numbers by rounding each fraction.

T: What does it mean to estimate?

S: We don't find the exact answer. → We find numbers at about the same value that are easier to work with. → We find an answer that is close but not exact. → If we estimate, it doesn't have to be exact.

T: Write $3\frac{1}{5} + 4\frac{8}{9}$. Let's estimate the sum.

T: Round $3\frac{1}{5}$. Think about benchmark numbers.

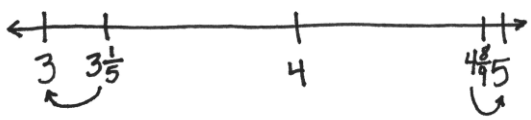
S: $3\frac{1}{5}$ is close to 3. → It's a little bit more than 3. → It's $\frac{1}{5}$ more than 3. → I round down to 3.

T: Round $4\frac{8}{9}$.

S: $4\frac{8}{9}$ is close to 5. → It's a little less than 5. → It's $\frac{1}{9}$ less than 5. → I round up to 5.

T: Quickly show $3\frac{1}{5}$ and $4\frac{8}{9}$ on a number line with endpoints at 3 and 5, only marking whole numbers and the two addends.

S: (Construct and label a number line.)



T: Notice how close the mixed numbers are to the rounded numbers. What is the estimated sum?

S: 3 + 5 = 8. Eight is our estimate.

T: What if we were to estimate the difference?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If necessary, present the visual of a number line to support students working below grade level as they round mixed numbers.

S: We would still round to 3 and 5 and subtract 3 from 5. The difference of $4\frac{8}{9}$ and $3\frac{1}{5}$ is about 2.

T: Talk to your partner: Will the actual difference be a little more than 2 or a little less than 2?

S: A little less because you can see from the number line that the difference is greater when we rounded. → A little less because the number line shows that the distance between $3\frac{1}{5}$ and $4\frac{8}{9}$ is less than 2.

Problem 2: Round two mixed numbers to the nearest half or whole number, and then find the sum.

T: Write $8\frac{9}{10} + 2\frac{4}{8}$. What's $8\frac{9}{10}$ rounded to the nearest one?

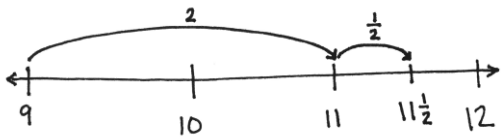
S: 9.

T: How about $2\frac{4}{8}$? Do we need to round $2\frac{4}{8}$?

S: No. $2\frac{4}{8}$ is the same as $2\frac{1}{2}$. Can I keep it as $2\frac{1}{2}$?

T: Yes. $9 + 2\frac{1}{2}$ is …?

S: It's just 11 and then another half—$11\frac{1}{2}$. → Well, I can think of 9 on a number line, and then I can picture adding two and a half more. Two more makes 11. → $11 + \frac{1}{2} = 11\frac{1}{2}$.



T: Why is your estimate greater than the actual sum? Talk to your partner.

S: It's greater because we rounded up $8\frac{9}{10}$. We made it bigger. → Our estimate is greater than the actual amount because we rounded 9 tenths up to 1. → We didn't round $2\frac{4}{8}$ at all, but we did round up $8\frac{9}{10}$ by $\frac{1}{10}$, so our actual answer will be $\frac{1}{10}$ less than our estimate.

Problem 3: Estimate the difference of two fractions greater than 1.

T: Write $\frac{15}{4}$ and $\frac{22}{7}$. What do you notice about these fractions?

S: They have different units. → They are more than 1.

T: Go ahead and convert each to a mixed number.

S: $\frac{15}{4} = \left(3 \times \frac{4}{4}\right) + \frac{3}{4} = 3\frac{3}{4}$, and $\frac{22}{7} = \left(3 \times \frac{7}{7}\right) + \frac{1}{7} = 3\frac{1}{7}$.

T: Round $3\frac{1}{7}$ to the nearest one. Round $3\frac{3}{4}$ to the nearest one.

MP.4

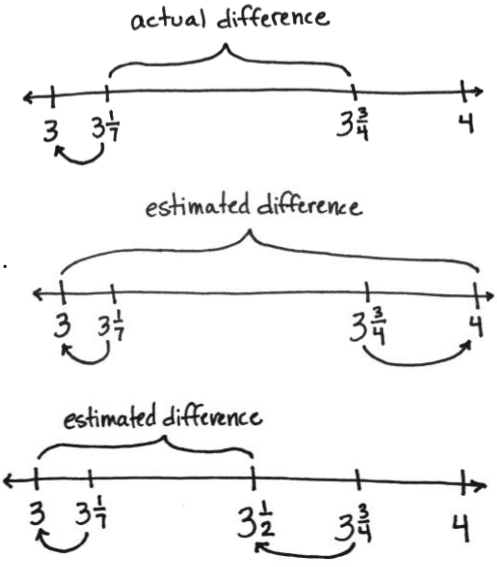
S: 3. 4.

T: 4 – 3?

S: 1.

T: How else could you round to be more precise?

S: I could round $3\frac{3}{4}$ to $3\frac{1}{2}$ and $3\frac{1}{7}$ to 3. The estimated difference would be $\frac{1}{2}$.



T: Discuss with your partner. Which estimate is closer?

MP.4

S: One-half is closer. I know that because I took a little away from $3\frac{3}{4}$ to get $3\frac{1}{2}$ and a little away from $3\frac{1}{7}$ to get 3. Taking away a little from each means the difference is almost the same. I can see that on a number line.

To verify that final statement (or to make it), take a string, and stretch it from $3\frac{1}{7}$ to $3\frac{3}{4}$ on the number line. Then, without adjusting its length at all, move it to the left to now match 3 and 3 and a half. The difference is about the same.

Problem 4: Use benchmark numbers or mental math to estimate the sum and difference of two mixed numbers.

T: (Write $18\frac{7}{12}$ and $17\frac{3}{8}$.) Estimate the sum using benchmark numbers or mental math. Discuss your strategy with a partner.

S: $18\frac{7}{12}$ is close to $18\frac{1}{2}$, and $17\frac{3}{8}$ is close to $17\frac{1}{2}$. I can add the whole numbers first to get 35. 2 halves make 1. 35 and 1 is 36. → $18\frac{1}{2} + 17\frac{1}{2} = 35 + \left(\frac{1}{2}+\frac{1}{2}\right) = 36$. The sum is about 36.

T: Now, estimate the difference of the same two numbers.

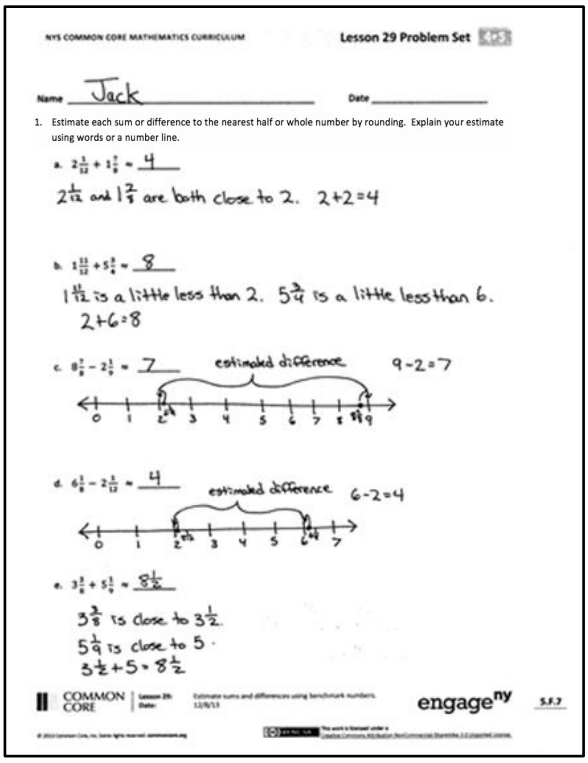
S: I can round to 19 and 17. → But that's rounding up and down, which makes the estimated difference bigger. Remember the example in the last problem? → I can just count up from $17\frac{1}{2}$ to $18\frac{1}{2}$, one. → There are two halves between them. Two halves make 1.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Scaffold finding the sum and difference of $18\frac{7}{12}$ and $17\frac{3}{8}$ for students working below grade level by chunking. First, isolate the fractions. Guide students to find the benchmark closest to $\frac{7}{12}$. Then, reintroduce the whole numbers.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 29 Problem Set

Name Jack Date

1. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

a. $2\frac{1}{12} + 1\frac{5}{8} = 4$

$2\frac{1}{12}$ and $1\frac{5}{8}$ are both close to 2. 2+2=4

b. $1\frac{11}{12} + 5\frac{3}{4} = 8$

$1\frac{11}{12}$ is a little less than 2. $5\frac{3}{4}$ is a little less than 6. 2+6=8

c. $8\frac{1}{8} - 2\frac{1}{9} = 7$ estimated difference 9-2=7

d. $6\frac{1}{8} - 2\frac{1}{12} = 4$ estimated difference 6-2=4

e. $3\frac{3}{8} + 5\frac{1}{9} = 8\frac{1}{2}$

$3\frac{3}{8}$ is close to $3\frac{1}{2}$.
$5\frac{1}{9}$ is close to 5.
$3\frac{1}{2}+5=8\frac{1}{2}$

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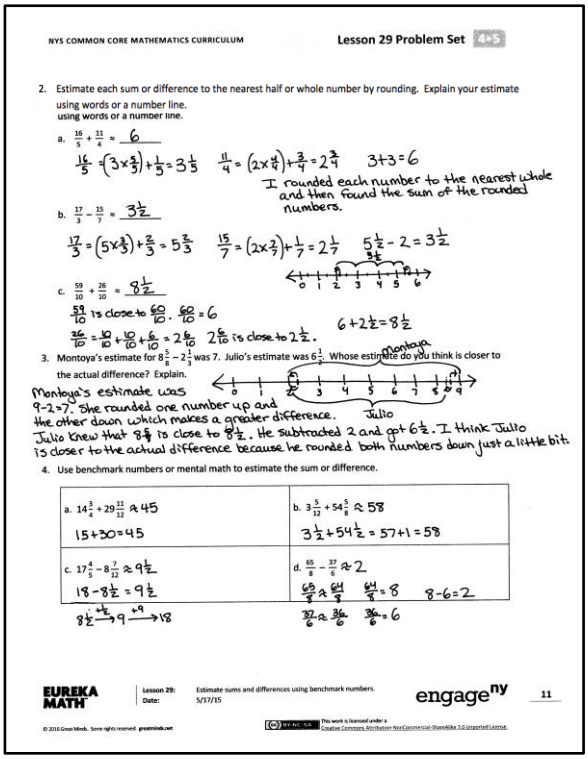
Student Debrief (10 minutes)

Lesson Objective: Estimate sums and differences using benchmark numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Student Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 29 Problem Set 4•5

2. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

a. $\frac{16}{5} + \frac{11}{4} \approx$ 6

$\frac{16}{5} = (3 \times \frac{3}{5}) + \frac{1}{5} = 3\frac{1}{5}$ $\frac{11}{4} = (2 \times \frac{4}{4}) + \frac{3}{4} = 2\frac{3}{4}$ 3+3=6

I rounded each number to the nearest whole and then found the sum of the rounded numbers.

b. $\frac{17}{3} - \frac{15}{7} \approx$ $3\frac{1}{2}$

$\frac{17}{3} = (5 \times \frac{3}{3}) + \frac{2}{3} = 5\frac{2}{3}$ $\frac{15}{7} = (2 \times \frac{7}{7}) + \frac{1}{7} = 2\frac{1}{7}$ $5\frac{1}{2} - 2 = 3\frac{1}{2}$

c. $\frac{59}{10} + \frac{26}{10} \approx$ $8\frac{1}{2}$

$\frac{59}{10}$ is close to $\frac{60}{10}$. $\frac{60}{10} = 6$

$\frac{26}{10} = \frac{10}{10} + \frac{10}{10} + \frac{6}{10} = 2\frac{6}{10}$ $2\frac{6}{10}$ is close to $2\frac{1}{2}$.

$6 + 2\frac{1}{2} = 8\frac{1}{2}$

3. Montoya's estimate for $8\frac{5}{8} - 2\frac{1}{3}$ was 7. Julio's estimate was $6\frac{1}{2}$. Whose estimate do you think is closer to the actual difference? Explain.

Montoya's estimate was 9-2=7. She rounded one number up and the other down which makes a greater difference. Julio knew that $8\frac{5}{8}$ is close to $8\frac{1}{2}$. He subtracted 2 and got $6\frac{1}{2}$. I think Julio is closer to the actual difference because he rounded both numbers down just a little bit.

4. Use benchmark numbers or mental math to estimate the sum or difference.

a. $14\frac{3}{4} + 29\frac{11}{12} \approx 45$ $15+30=45$	b. $3\frac{5}{12} + 54\frac{5}{8} \approx 58$ $3\frac{1}{2}+54\frac{1}{2}=57+1=58$
c. $17\frac{4}{5} - 8\frac{7}{12} \approx 9\frac{1}{2}$ $18-8\frac{1}{2}=9\frac{1}{2}$ $8\frac{1}{2} \xrightarrow{+\frac{1}{2}} 9 \xrightarrow{+9} 18$	d. $\frac{65}{8} - \frac{37}{6} \approx 2$ $\frac{65}{8} \approx \frac{64}{8}$ $\frac{64}{8}=8$ $8-6=2$ $\frac{37}{6} \approx \frac{36}{6}$ $\frac{36}{6}=6$

EUREKA MATH Lesson 29: Estimate sums and differences using benchmark numbers. Date: 5/17/15 engage^ny 11

- In Problems 1(a) and 1(b), all fractions could be rounded up or down by one unit fraction. Which of the two estimates is closer to the actual amount?
- If one of the two fractions in Problem 1(a) was rounded down to half, the estimate would be more accurate than rounding both to the nearest one. How do you decide which fraction rounds up and which one rounds down?
- Did your partner have the same estimates as you in Problem 2? Why or why not? Whose estimates are closer to the actual answers?
- Think about Problem 3. When would estimates need to be very close to the actual answer? When might estimates be acceptable if the numbers were rounded to the closest whole number?
- Some students estimated 45 or $44\frac{3}{4}$ for Problem 4(a). Some students estimated 9 or $9\frac{1}{2}$ for Problem 4(c). Which answer for each problem is more reasonable? How does someone determine how accurate the answer is?
- What prior knowledge about fractions did you use as you completed the problems in the Problem Set?
- What tools did you use to help you estimate?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

1. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

a. $2\frac{1}{12} + 1\frac{7}{8} \approx$ _____

b. $1\frac{11}{12} + 5\frac{3}{4} \approx$ _____

c. $8\frac{7}{8} - 2\frac{1}{9} \approx$ _____

d. $6\frac{1}{8} - 2\frac{1}{12} \approx$ _____

e. $3\frac{3}{8} + 5\frac{1}{9} \approx$ _____

2. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

   a. $\frac{16}{5} + \frac{11}{4} \approx$ _____

   b. $\frac{17}{3} - \frac{15}{7} \approx$ _____

   c. $\frac{59}{10} + \frac{26}{10} \approx$ _____

3. Montoya's estimate for $8\frac{5}{8} - 2\frac{1}{3}$ was 7. Julio's estimate was $6\frac{1}{2}$. Whose estimate do you think is closer to the actual difference? Explain.

4. Use benchmark numbers or mental math to estimate the sum or difference.

a. $14\frac{3}{4} + 29\frac{11}{12}$	b. $3\frac{5}{12} + 54\frac{5}{8}$
c. $17\frac{4}{5} - 8\frac{7}{12}$	d. $\frac{65}{8} - \frac{37}{6}$

Name _____ Date _____

Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

1. $2\frac{9}{10} + 2\frac{1}{4} \approx$ _____

2. $11\frac{8}{9} - 3\frac{3}{8} \approx$ _____

Name _____ Date _____

1. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

a. $3\frac{1}{10} + 1\frac{3}{4} \approx$ _____

b. $2\frac{9}{10} + 4\frac{4}{5} \approx$ _____

c. $9\frac{9}{10} - 5\frac{1}{5} \approx$ _____

d. $4\frac{1}{9} - 1\frac{1}{10} \approx$ _____

e. $6\frac{3}{12} + 5\frac{1}{9} \approx$ _____

2. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

   a. $\frac{16}{3} + \frac{17}{8} \approx$ _____

   b. $\frac{17}{3} - \frac{15}{4} \approx$ _____

   c. $\frac{57}{8} + \frac{26}{8} \approx$ _____

3. Gina's estimate for $7\frac{5}{8} - 2\frac{1}{2}$ was 5. Dominick's estimate was $5\frac{1}{2}$. Whose estimate do you think is closer to the actual difference? Explain.

4. Use benchmark numbers or mental math to estimate the sum or difference.

a. $10\frac{3}{4} + 12\frac{11}{12}$	b. $2\frac{7}{10} + 23\frac{3}{8}$
c. $15\frac{9}{12} - 8\frac{11}{12}$	d. $\frac{56}{7} - \frac{31}{8}$